The Pervasive Hubble Expansion of the Universe

by
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Introduction. The treatise described in this paper is based on one assumption. Namely, if an aether fluid exists then its fluid elements must recede from each other in accord with the Hubble law. Or one must assume that the distances, d(t) between the galaxies of the universe and the fluid elements of the aether at any time, t obey Hubble's law

$$d(t) = \gamma(t)d(0) \tag{I.1}$$

where $\gamma(t) = 1+t/\mathbf{T}$, t is time, $\mathbf{T}$ is the present Hubble age, and d(0) are the distances at the present time, t = 0. It is the only expansion which satisfies necessary condition that the recessional motion of the galaxies and the fluid elements of the aether are the same on a large scale relative to all galaxies of the universe. The Hubble law is usually given in the form $d(t)/dt = d(0)/\mathbf{T}$ and $1/\mathbf{T}$ is called the Hubble constant, H. One usually determines H by giving d(t)/dt in km/sec and d(0) in mega parsec. By (I.1) one may also write (I.1) as $d(t)/dt = d(t)/\mathbf{T}\gamma(t)$.

Light propagation in such an expanding aether has never been investigated. Likely because the analytical problems involved seam insuperable. For one thing one would have to know the mathematical description of light superimposed on the expanding aether.

An exact solution [1] of Euler's equations for the monatomic adiabatic ideal fluid bypasses the insuperable problems. The solution is an intrinsic part of the Hubble law. It determines in general the nonlinear interactions between the Hubble expansion of this fluid and any fluid motions superimposed on it. The nonlinear interactions are identical for all superimposed motions. Moreover, they are stated precisely without any knowledge of the mathematical description of the superimposed motions. One finds that Hubble's law as it now exists is only a small part of an extended Hubble law, called the pervasive Hubble law, which resolves outstanding questions and paradoxes that have puzzled and irritated scientists for many years.

Section 1. The solution [1] relates two fluid motions in adiabatic monatomic ideal fluids. One motion has fluid density, ρ and velocity, v_i with independent variables, x_1, x_2, x_3, t, written as x's,t. The other has fluid density, ρ′ and velocity, v_i' with independent variables, X_1, X_2, X_3, T, written as X's,T. The coordinates x's and X's are coaxial coordinates with origin, O. The solution [1] is rewritten here with a slightly different notation as

$$\rho(x\text{'s},t) = [\gamma(t)]^{-3}\, \rho'(X\text{'s},T) \tag{1.1}$$

$$v_i(x\text{'s},t) = [\gamma(t)]^{-1}\, v_i'(X\text{'s},T) + v_i^*(x\text{'s},t) \tag{1.2}$$

$$v_i^*(x\text{'s},t) = x_i(t)/\mathbf{T}\gamma(t) \tag{1.3}$$

$$\rho^*(t) = [\gamma(t)]^{-3}\, \rho_o \tag{1.4}$$

$$x_n = \gamma(t)\, X_n \qquad (n = 1, 2, 3) \tag{1.5}$$

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$$t = \gamma(t)\, T \qquad (1.6)$$

$$\gamma(t) = 1 + t/\mathbf{T} \qquad (1.7)$$

The fluid motion with density ρ^* and velocity, v_i^* is the undisturbed aether expansion, R, $x_i(t)$ is the radius vectors to a fluid element of R relative to O, ρ_o is the density of the stationary fluid, and $\mathbf{T}$ is the present value of the Hubble age. Equations(1.3) is in the form of Hubble's law (I.3). For brevity the parenthesis on γ is omitted when the argument is t. By (1.6) and (1.7) one finds that the function, γ can also be written in terms of T as $\gamma(T) = (1-T/\mathbf{T})^{-1}$.

The fluid density, ρ and velocity, v_i are solutions of Euler's equations for the adiabatic monatomic ideal fluid, namely,

$$Dv_i(x's,t)/Dt = - [\rho(x's,t)]^{-1}\, \partial p(x's,t)/\partial x_i \qquad (1.8)$$

$$D[\ln\rho(x's,t)]/Dt = - \partial v_i(x's,t)/\partial x_i \qquad (1.9)$$

Since the ideal fluid is adiabatic and monatomic the pressure, p must be given by

$$p(x's,t) = (3/5)\rho_o \hat{c}^2[\rho(x's,t)/\rho_o]^{5/3} \qquad (1.10)$$

The fluid density, ρ' and velocity, v_i' are solutions of Euler's equations for the same ideal fluid. These equations are

$$Dv_i'(X's,T)/DT = - [\rho'(X's,T)]^{-1}\, \partial p'(X's,T)/\partial X_i \qquad (1.11)$$

$$D(\ln\rho'(X's,T))/DT = -\partial v_i'(X's,T)/\partial X_i \qquad (1.12)$$

The pressure, p' for the same ideal fluid must be given by

$$p'(X's,T) = 3/5\rho_o \hat{c}^2[\rho'(X's,T)]/\rho_o)^{5/3} \qquad (1.13)$$

where $\partial/\partial x_i$ and $\partial/\partial X_i$ are gradient operators, and D/Dt and D/DT are Lagrangian operators.

By (I.1) all reference frames, x_1', x_2', x_3' with origins, O' coinciding with a differential fluid element of the undisturbed R are inertial frames. One shows readily that in these reference frames the form of the (1.1) to (1.7) is the same, and the velocities of the differential fluid elements of R relative to O' are isotropic. These frames are called "isotropic reference frames".

Section 2. The proof of solution [1] is repeated here using notations adopted in this paper. One proves that if ρ and v_i satisfy (1.8) and (1.9) then by (1.1) to (1.7), (1.10), and (1.13) ρ' and v_i' satisfy (1.11) and (1.12) and visa versa. In the proof for sake of clarity one leaves out the parentheses with the independent variables on v_i, ρ, v_i^*, ρ^*, and v_i', and ρ'. The Lagrangian operator, D/Dt in terms of partial derivatives, $\partial/\partial t$ and gradient operators, $\partial/\partial x_i$, is given by

$$D/Dt \equiv \partial/\partial t + v_j\partial/\partial x_j \tag{2.1}$$

See, for example, [17, art 37]. This operator on v_i^* in (1.2) yields

$$Dv_i^*/Dt = \partial v_i^*/\partial t + v_j\partial v_i^*/\partial x_j \tag{2.2}$$

Note that $\partial v_i^*/\partial x_j = \delta_{ji}/\mathrm{T}\gamma$ where δ_{ji} is the item factor. By (1.3) and (2.2) one obtains

$$Dv_i^*/Dt = (v_i - v_i^*)/\mathrm{T}\gamma \tag{2.3}$$

One notes by (1.2) that in regions where v_i' vanishes v_i equals v_i^*, and Dv_i^*/Dt is identically zero in these regions where. Therefore, any differential fluid element of R at the radius vector, $x_i(t)$ at time, t moves with constant inertial velocity, which is given either by, $v_i^* = x_i(t)/\mathrm{T}\gamma$ or by

$$v_i^* = x_i(0)/\mathrm{T} \tag{2.4}$$

where $x_i(0)$ is its position at time $t = 0$. Apparently, the undisturbed aether expansion, R is in accord with Euler's nonlinear equations for this fluid.

For the left side of (1.8) one writes

$$Dv_i/Dt = \partial v_i/\partial t + \partial v_j\partial v_i/\partial x_j \tag{2.5}$$

By (1.2)

$$Dv_i/Dt = D(\gamma^{-1} v_i')/Dt + Dv_i^*/Dt \tag{2.6}$$

By (2.6), (1.7), and (2.3)

$$Dv_i/Dt = - v_i'/\mathrm{T}\gamma^2 + \gamma^{-1} Dv_i'/Dt + (v_i - v_i^*)/\mathrm{T}\gamma \tag{2.7}$$

By (1.2) the first and last term on the right side of (2.7) cancel and (2.7) becomes

$$Dv_i/Dt = \gamma^{-1}Dv_i'/Dt \tag{2.8}$$

By (2.1) and (2.8)

$$Dv_i/Dt = \gamma^{-1} (\partial v_i'/\partial t + v_j\, \partial v_i'/\partial x_j) \tag{2.9}$$

Since x_i remains constant in derivatives, $\partial/\partial t$, but by (A.5) T and X_i do not, $\partial/\partial t$ can be written

$$\partial/\partial t = (\partial/\partial T)\partial T/\partial t + (\partial/\partial X_i)\partial X_i/\partial t \tag{2.10}$$

Since t remains constant in the gradient $\partial/\partial x_j$, by (1.6) T also remains constant, but X_j does not, hence $\partial/\partial x_j$ can be written

$$\partial/\partial x_j = (\partial/\partial X_j)\partial X_i/\partial x_i \tag{2.11}$$

By (1.2), (2.10), and (2.11) one obtains for (2.9)

$$Dv_i/Dt = \gamma^{-1}[(\partial v_i'/\partial T)\,\partial T/\partial t + (\partial v_i'/\partial X_i)\,\partial X_i/\partial t + + (\gamma^{-1} v_j' + v_j^*)(\partial v_i'/\partial X_j)\,\partial X_j/\partial x_j] \tag{2.12}$$

By (1.5) to (1.7) one obtains

$$\partial T/\partial t = \gamma^{-2} \tag{2.13}$$

$$\partial X_i/\partial t = - v_i^*/\gamma \tag{2.14}$$

$$\partial X_j/\partial x_j = \gamma^{-1} \tag{2.15}$$

By (2.12) to (2.15) one gets

$$Dv_i/Dt = \gamma^{-3}\,\partial v_i'/\partial T - \gamma^{-2} v_j^*\partial v_i'/\partial X_j + \gamma^{-3}\, v_j'\partial v_i'/\partial X_j + \gamma^{-2} v_j^*\partial v_i'/\partial X_j \tag{2.16}$$

The second and fourth term on the right side of (2.16) cancels, and

$$Dv_i/Dt = \gamma^{-3}\, Dv_i'/DT \tag{2.17}$$

where

$$Dv_i'/DT \equiv \partial v_i'/\partial T + v_j'\partial v_i'/\partial X_j \tag{2.18}$$

Thus, as a result of the expansion effects the acceleration, Dv_i/Dt of any differential fluid elements of the motion, ρ, v_i at x's,t is changed by the factor, γ^{-3} relative to the acceleration, Dv_i'/DT of the corresponding fluid element of the motion ρ', v_i' at X's,T.

By (1.1), (1.10), and (1.13) one finds

$$p = p'/\gamma^{5} \tag{2.19}$$

For the right side of (1.8) one obtains by(1.1), (2.19), and (2.11)

$$-\rho^{-1}\partial p/\partial x_i = -\gamma^{3}(\rho')^{-1}\,\gamma^{-5}[\partial p'/\partial X_i]\,\partial X_i/\partial x_i \tag{2.20}$$

By (2.15)

$$-\rho^{-1}\partial p/\partial x_i = -\gamma^{-3}(\rho')^{-1}\partial p'/\partial X_i \tag{2.21}$$

By (2.17) and (2.21) one observes that if (1.8) is satisfied then (1.11) is also satisfied and visa versa.

For the left side of (1.9) one obtains from (1.1)

$$D(\ln\rho)/Dt = D(\ln\rho')/Dt - 3/\mathbf{T}\gamma \qquad (2.22)$$

Or by (2.1)

$$D(\ln\rho)/Dt = \partial(\ln\rho')/\partial t + v_i\partial(\ln\rho')/\partial x_i - 3/\mathbf{T}\gamma \qquad (2.23)$$

For the first right hand term of (2.23) one finds by (2.10)

$$\partial(\ln\rho')/\partial t = [\partial(\ln\rho')/\partial T]\, \partial T/\partial t + [\partial(\ln\rho')/\partial X_i]\, \partial X_i/\partial t \qquad (2.24)$$

Or by (2.13) and (2.14)

$$\partial(\ln\rho')/\partial t = \gamma^{-2}\, \partial(\ln\rho')/\partial T - (v_i^*/\gamma)\, \partial(\ln\rho')/\partial X_i \qquad (2.25)$$

For the second right hand term of (2.23) one obtains by (1.2) and (2.11)

$$v_i\partial(\ln\rho')/\partial x_i = (\gamma^{-1}v_i' + v_i^*)[\partial(\ln\rho')/\partial X_i]\, \partial X_i/\partial x_i \qquad (2.26)$$

or by (2.15)

$$v_i\partial(\ln\rho')/\partial x_i = (\gamma^{-2}v_i')\, \partial(\ln\rho')/\partial X_i + (v_i^*/\gamma)\, \partial(\ln\rho')/\partial X_i \qquad (2.27)$$

When (2.25) and (2.27) are added the last terms in (2.25) and (2.27) cancel. Thus, (2.23) becomes

$$D(\ln\rho)/Dt = \gamma^{-2}\, D(\ln\rho')/DT - 3/\mathbf{T}\gamma \qquad (2.28)$$

For the right side of (1.9) one finds by (1.2)

$$-\partial v_i/\partial x_i = -\, \partial(\gamma^{-1}v_i')/\partial x_i - \partial v_i^*/\partial x_i \qquad (2.29)$$

By (2.11), (1.3), and the identity $\partial x_i/\partial x_i \equiv 3$ one obtains for (2.29)

$$-\partial v_i/\partial x_i = -\, \gamma^{-1}[\partial v_i'/\partial X_i]\, \partial X_i/\partial x_i - 3/\mathbf{T}\gamma \qquad (2.30)$$

By (2.15) one obtains for (2.30)

$$-\partial v_i/\partial x_i = -\, \gamma^{-2}\, \partial v_i'/\partial X_i - 3/\mathbf{T}\gamma \qquad (2.31)$$

By (2.28) and (2.31) if (1.9) is satisfied then (1.12) is also satisfied and visa versa. This completes the proof of the solution [1].

Section 3. One writes the (1.1) and (1.2) as

$$\rho(x\text{'s},t) - \rho^*(t) = \gamma^{-3}\, [\rho'(X\text{'s},T) - \rho_o] \qquad (3.1)$$

$$v_i(x\text{'s},t) - v_i^*(x\text{'s},t) = \gamma^{-1}[v_i'(X\text{'s},T)] \qquad (3.2)$$

The fluid motions on the left sides of (3.1) and (3.2) with fluid density, ρ - ρ^* and velocity v_i - v_i^* are fluid motions over and above expansion, R. The fluid motions in the brackets on the right side of (3.1) and (3.2) with density, ρ' - ρ_o and velocity, v_i' are fluid motions over and above the stationary aether. These two fluid motions are those referred in the introduction respectively by the capital letters N and M. They are by themselves not solutions of Euler's equations. They are solutions of (1.9) to (1.13) when they are superimposed on respectively R and the stationary fluid. Eq's (3.1), (3.2), and (1.3) to (1.7) is a one to one correspondence between N and M. They are equal at time, t = T = 0. At any other time N differs from M because of nonlinear interactions between N and R, called expansion effects. Quantity pairs associated with the two motions, like t and T, x's and X's, ρ-ρ^* and ρ'-ρ_o, v_i-v_i^* and v_i', etc. are called corresponding quantities.

By equation (I.1) all reference frames, x_1', x_2', x_3' with origins, O' coinciding with a differential fluid element of the R are inertial frames. One shows readily that in these reference frames the form of (1.1) to (1.7) is the same, and the velocities of the differential fluid elements of R relative to O' are isotropic. These frames are called isotropic frames.

In the analysis of N one uses two coordinate systems. The inertial coordinates, x's and expanding coordinates, $x_1^*(t)$, $x_2^*(t)$, $x_3^*(t)$, written as $x^*(t)$'s, defined as

$$x_n^*(t) = \gamma x_n \qquad (n = 1, 2, 3) \tag{3.3}$$

Apparently the $x^*(t)$'s and x's coordinates are coaxial, and coincide at time t = 0. By equation (I.1) each expanding coordinate point remains coincident with a fluid element of R. The galaxies of the universe which follow Hubble's law are at rest relative to the contiguous expanding coordinates, $x^*(t)$'s. Velocities relative to the $x^*(t)$'s and x's coordinates are called respectively local and inertial velocities.

To derive the expansion effects one notes that when **T** approaches infinity the N approaches M, R approaches the stationary fluid, and all expansion effects approach zero. Hence M equals N with all expansion effects removed. Therefore, the expansion effects are the difference between quantities associated with N evaluated at (x's,t) and corresponding quantities associated with M evaluated at (X's,T). One finds that all expansion effects are in the form of multiplying factors containing the Hubble function, γ. The expansion effects are as follows.

Corresponding differential volume elements, $d\upsilon$(x's,t) or $dx_1dx_2dx_3$ for N and $d\upsilon'$(X's,T) or $dX_1dX_2dX_3$ for M are by (1.5) related as

$$d\upsilon(x's,t) = \gamma^3\, d\upsilon'(X's,T) \tag{3.4}$$

Thus, differential volume element of N must have γ^3 as a multiplying factor.

By(1.5) the radius vector, $x_i(t)$ to $d\upsilon$(x's,t) at time, t is related to the corresponding radius vector, $X_i(T)$ to $d\upsilon'$(X's,T) at the corresponding time, T as

$$x_i(t) = \gamma\, X_i(T) \tag{3.5}$$

Thus, the radius vectors to differential volume elements of N must have γ as a multiplying factor.

If $L_i'(T)$ is a vector between the positions, $[X_i(T)]_1$ and $[X_i(T)]_2$ of any two fluid elements of any M motion at time, T then by(3.5) the vector $L_i(t)$ between the corresponding positions, $[x_i(t)]_1$ and $[x_1(t)]_2$ of the N motion at time, t is given by

$$L_i(t) = \gamma\, L_i'(T) \tag{3.6}$$

Thus, any vector, $L_i(t)$ for N must have γ as a multiplying factor. Moreover, any length or dimension associated with M at time, T is for N a corresponding parallel length or parallel dimension longer by the factor, γ. Thus, any length or dimension of N must have γ as a multiplying factor.

Corresponding volumes, V(t) at time, t and $V'(T)$ at time T are defined as closed volumes filled with any number of corresponding volume elements $d\upsilon$ and $d\upsilon'$. By (1.5) and (3.4) they are related as

$$V(x\text{'s},t) = \gamma^3\, V'(X\text{'s},T) \tag{3.7}$$

Thus, any volume associated with N must have γ^3 as a multiplying factor.By (3.4) and (3.7) the volumes may range in size from differential volumes to all of space. By (3.6) the enlargement of V(t) at time, t relative to $V'(T)$ at the earlier time, T occurs without relative rotation.

The mass, m(t) within V(t) at time, t of any N motion or any of its parts is defined as

$$m(t) = \iiint_{V(t)}[\rho(x\text{'s},t) - \rho^*(t)]dx_1dx_2dx_3 \tag{3.8}$$

The mass, $m'(T)$ within $V'(T)$ at time T of the corresponding M motion is defined as

$$m'(T) = \iiint_{V'(T)}[\rho'(X\text{'s},T) - \rho_o]dX_1dX_2dX_3 \tag{3.9}$$

For each mass element, $[\rho(x\text{'s},t)-\rho^*(t)]dx_1dx_2dx_3$ in V(t) there is by (1.1) and (1.5) an equal corresponding mass element, $[\rho'(X\text{'s},T)-\rho_o]dX_1dX_2dX_3$ in $V'(T)$. Hence,

$$m(t) = m'(T) \tag{3.10}$$

Thus, m(t) has no multiplying factor containing the Hbble function, γ. One notes that (3.10) follows from (3.8) without $\rho^*(t)$ and (3.9) without ρ_o. Thus (2.7) is valid also for N superimposed on R and M superimposed on the stationary fluid. The expansion effects do not by themselves introduce any change in mass. They merely change any differential time span, dT for any mass changes in M to a corresponding longer time span, dt for an equal mass change in N or N superimposed on R. By (1.6) the two differential time elements are related as, $dT = dt/\gamma^2$.

The radius vector, $H_i(t)$ to the mass center of m(t) at time, t is defined as

$$m(t)H_i(t) = \iiint_{V(t)}x_i(t)[\rho(x\text{'s},t) - \rho^*(t)]dx_1dx_2dx_3 \tag{3.11}$$

The radius vector, $H_i'(T)$ to the mass center of the corresponding $m'(T)$ at time, T is

$$m'(T)H_i'(T) = \iiint_{V'(T)} X_i(T)[\rho'(X's,T) - \rho_o]dX_1dX_2dX_3 \qquad (3.12)$$

By (3.4), (3.5), (1.5), and (3.10)

$$H_i(t) = \gamma\, H_i'(T) \qquad (3.13)$$

Thus, radius vectors to mass centers of N must have γ as a multiplying factor.

The inertial mass center velocity, $u_i(t)$ of any given N motion and any of its parts in V(t) at time, t and the mass center velocity, $u_i'(T)$ of the M motion in the corresponding volume, $V'(T)$ at time, T, are respectively given by

$$u_i(t) = dH_i(t)/dt \qquad (3.14)$$

and

$$u_i'(T) = dH_i'(T)/dT \qquad (3.15)$$

By (3.13) to (3.15) one obtains

$$u_i(t) - H_i(t)/\mathbf{T}\gamma = \gamma^{-1}u_i'(T) \qquad (3.16)$$

The derivative with respect to t of (3.16) yields five terms. By (3.16) three of these cancel, and one obtains

$$du_i(t)/dt = \gamma^{-3}\, du_i'(T)/dT \qquad (3.17)$$

or by (3,14) and (3.15)

$$d^2H_i(t)/dt^2 = \gamma^{-3}\, d^2H_i'(T)/dT^2 \qquad (3.18)$$

By (1.3) the term, $H_i(t)/\mathbf{T}\gamma$ in (3.16) is the velocity of the undisturbed aether expansion, R at $H_i(t)$. It is also the velocity relative to O of the expanding coordinate point which coinsides with $H_i(t)$. Therefore, the left side of (3.16) is the local mass center velocity, $w_i(t)$ of N motions. Thus, local mass center velocities of N relative to corresponding velocities of M is given by

$$w_i(t) = \gamma^{-1}u_i'(T) \qquad (3.19)$$

Thus, local mass center velocities for N must have γ^{-1} as a multipluing factor.

The local kinetic energy, $[E_k(t)]_L$ of N or any of its parts within a volume, V(t) is given by

$$[E_k(t)]_L = \iiint_{V(t)} (1/2)[\rho(x's,t) - \rho^*(t)][v_i(x's,t) - v_i^*(x's,t)]^2dx_1dx_2dx_3 \qquad (3.20)$$

The kinetic energy energy, $E_k'(T)$ of the M motion in the corresponding volume, $V'(T)$ is

$$E_k'(T) = \iiint_{V'(T)} (1/2)[\rho'(X's,T) - \rho_o][v_i'(X's,T)]^2 dX_1 dX_2 dX_3 \quad (3.21)$$

By (3.1), (3.2), and (1.3) to (1.7)

$$[E_k(t)]_L = \gamma^{-2} E_k'(T) \quad (3.22)$$

If $\mathcal{P}(x's,t)$ and $\mathcal{P}^*(x's,t)$ are the potential energies per unit volume for the ρ, v_i motion and the undisturbed expansion, ρ^*, v_i^* respectively, the potential energy, $P(t)$ of any given N motion within a volume, $V(t)$ is

$$P(t) = \iiint_{V(t)} [\mathcal{P}(x's,t) - \mathcal{P}^*(x's,t)] dx_1 dx_2 dx_3 \quad (3.23)$$

If $\mathcal{P}'(X's,T)$ and $\mathcal{P}_o$ are the potential energies per unit volume for the ρ', v_i' motion and the stationary fluid respectively, the potential energy, $P'(T)$ of the corresponding M motion within the corresponding volume, $V'(T)$ is

$$P'(T) = \iiint_{V'(T)} [\mathcal{P}'(X's,T) - \mathcal{P}_o] dX_1 dX_2 dX_3 \quad (3.24)$$

For the adiabatic monatomic ideal fluid the potential energy per unit volume equals 3/2 times the fluid pressure. Hence, by (2.19)

$$\mathcal{P}(x's,t) = \gamma^{-5} \mathcal{P}'(X's,T) \quad (3.25)$$

and

$$\mathcal{P}^*(x's,t) = \gamma^{-5} \mathcal{P}_o \quad (3.26)$$

Therefore, the integrand in (3.21) equals γ^{-2} times the integrand in (3.22), and

$$P(t) = \gamma^{-2} P'(T) \quad (3.27)$$

The total local energy, $[E(t)]_L$ of N motions in $V(t)$ and the total energy, $E'(T)$ of M motions in $V'(T)$ are the sums of their kinetic and the potential energies, or

$$[E(t)]_L = \gamma^{-2} E'(T) \quad (3.28)$$

Thus, local energies for N must have γ^{-2} as a multiplying factor.

By (1.6) and (1.7) one obtains

$$t - T = t^2/\mathbf{T}\gamma \quad (3.29)$$

By (3.29) one notes that t is always a later time than T. Thus any characteristic event in M at time, T is for N delayed by $t^2/\mathbf{T}\gamma$. For example an event for M may be the compleation of one orbital period or several orbital periods. The compleation of the corresponding periods for N is by (1.6) and (1.7) longer by the factor, γ. Moreover, if one subtract the delay, $t^2/\mathbf{T}\gamma$ from t one obtains solar ephemeris time, t_s given by

$$t_s = t/\gamma \qquad (3.30)$$

These and other expansion effects derived in Sections 4a to 4f are together with Hubble's law (I.1) the pervasive Hubble's law. The pervasive Hubble law apply without any limitations relate all M, over and above the stationary aether, and the initially identical N over and above R.

In Sections 4a to 4f the above expansion effects are used to find evidences in support of the thesis that *all forms of matter, fields, and light are various fluid motions, N over and above R.* Solutions of present theories of physics do not have multiplying factors containing the function, γ or some other functions containing $\mathbf{T}$. Hence these solutions do not account for the expansion effects. The only sensible assumption is that they are correct except for these deficiencies. Therefore, if the above thesis be correct solutions of present theories of physics must be N motions in the limit when $\mathbf{T}$ approaches infinity and all expansion effects approach zero. Or solutions of present theories of physics must be M motions. A corollary of the above thesis is therefore that *solutions of present theories of physics are M motions over and above the stationary aether fluid*. To remove the deficiencies one simply multiplies quantities associated with solutions of present theories of physics by the appropriate multiplying factors containing the Hubble function, γ. If the corollary be correct there must be discrepancies between solutions of present theories and observations and unresolved outstanding questions. Several such discrepancies exist and are investigated in Sections 4a to 4f and also in three Appendices.

Section 4a. One studies here the propagation of a phase of light emitted at the origin, O at time, t = 0. One assumes that it remains remote from large gravitating bodies. By present theory, or for M, the light wave length is invariant with time, T. The wave length, $\lambda(t)$ for N is by (3.36) given by

$$\lambda(t) = \gamma\lambda(0) \qquad (4a.1)$$

For M the light velocity remains invariant with time, T equal to the constant, $\hat{c}$. By (3.19) for N the local light velocity equals

$$c_i(t) = \hat{c}/\gamma \qquad (4a.2)$$

One notes in passing that $c_i(t)$ is also equal to $(dp/d\rho)^{1/2}$ in the limit as ρ approaches ρ^*. This is the light velocity relative to the undisturbed expanding aether. Thus, as assumed above, $c_i(t)$ a local light velocity.

For N the inertial phase velocity, $C_i(t)$ for a phase at the radius vector, $x_i(t)$ relative to O at time, t is obtained by adding the expansion velocity, v_i^* at $x_i(t)$, or by (1.3)

$$Ci(t) = c_i(t) + x_i/(t+\mathbf{T}) \qquad (4a.3)$$

which can also be written

$$x_i(t) = (t+\tau)C_i(t) - \hat{c}\tau \quad (4a.4)$$

Since $x_i(t)$ is the position of the light phase at time, t

$$dx_i(t)/dt = C_i(t) \quad (4a.5)$$

Hence by (4a.4) and (4a.5)

$$C_i(t) = C_i(t) + (t+\tau)dC_i(t)/dt \quad (4a.6)$$

or for any given phase of light

$$dC_i(t)/dt = 0 \quad (4a.7)$$

By (4a.3) the inertial light velocity, C_i for a phase which is at any inertial coordinate point, x_i at time t = 0 is given by

$$C_i = \hat{c}_i + x_i/\tau \quad (4a.8)$$

Thus, C_i is a larger constant for a phase ahead than for a phase behind.

For the phase emitted a the origin, O at time t = 0, C_i remains invariant with time, t, and is given by

$$C_i = \hat{c}_i \quad (4a.9)$$

By (4a.1) and (4a.9) the frequency, f(t) observed for this phase at an inertial coordinate point with radius vector, x_i relative to O, called the inertial frequency, is given by

$$f(t) = f(0)/\gamma \quad (4a.10)$$

Passing from the inertial coordinate point x_i to a new isotropic reference frame with origin, O' which coincides with x_i at time, t, one finds that the light velocity, $\hat{c}_i$ changes to $\hat{c}_i/\gamma$, the wave length $\gamma\lambda(0)$ remains the same, and the frequency, f_o/γ changes to f(t) given by

$$f(t) = f(0)/\gamma^2 \quad (4a.11)$$

By Planck's law its local photonic energy, $[E_p(t)]_L$ is given by

$$[E_p(t)]_L = h_o f(0)/\gamma^2 \quad (4a.12)$$

The photonic energy, $[E_p(t)]_I$ observed at an inertial point coinciding with O' is given by

$$[E_p(t)]_I = h_o f(0)/\gamma \quad (4a.13)$$

By present theory, or for M, m′(T) in (3.10) remains invariant with time, T when it moves from O to a point with radius vector, X_i relative to O. Therefore, the mass, m(t) for any N remains invariant with time, t when it moves from O to a point with radius vector, x_i relative to O. Or

$$m(t) = \underline{m} \qquad (4a.14)$$

where $\underline{m}$ is a constant. By (4a.12) and (4a.13) the photonic energy changes by the factor, $1/\gamma$ when passing from an inertial point to a coinciding O'. Hence the mass, m(t) observed at an inertial point must change to m'(t) when observed at a coinciding O', or

$$m'(t) = \underline{m}/\gamma \qquad (4a.15)$$

One assumes that Einstein's law for N at any time, t is that any mass, m(t) has energy equal to $m(t)[c_i(t)]^2$. Thus,

$$E'_p(t) = \underline{m}\hat{c}/\gamma^3 \qquad (4a.16)$$

By (4a.12) and (4a.16) the Planck constant, h_o must be replaced by the Planck function, h(t) given by

$$h(t) = h_o/\gamma \qquad (4a.17)$$

A very important consequence of (4a.17) is as follows. The local energy observed of any N at any origin, O' of a pervasive reference frame is given by (3.28). By present theory E′(T) is invariant with time, T, or E′(T) equals the constant, $[E(0)]_L$. Thus,

$$[E(t)]_L = \gamma^{-2} [E(0)]_L \qquad (4a.18)$$

The local enery levels $[E_a(t)]_L$ and their differences $\Delta[E_a(t)]_L$ of any atom at O' are by (4a.18) related as $[E_a(t)]_L = \gamma^{-2} [E_a(0)]_L$ and $\Delta[E_a(t)]_L = \gamma^{-2} \Delta[E_a(0)]_L$. Therefore, by Planck's law for N the frequency, $f_e(t)$ emitted as the atom passes from any $[E_a(t)]_L$ to $[E_a(t)]_L - \Delta[E_a(t)]_L$ is given by

$$\Delta[E_a(t)]_L = h_o/\gamma \; f_e(t) \qquad (4a.19)$$

By (4a.18) and (4a.19) $f_e(t)$ equals a constant divided by γ or

$$f_e(t) = f_e(0)/\gamma \qquad (4a.20)$$

The inertial light velocity in any isotropic reference frame with origin, O' at time, t is $\hat{c}/\gamma$. Hence the emitted wave length, $\lambda_e(t)$ at any origin, O' is invariant with, t, or it equals

$$\lambda_e(t) = \underline{\lambda} \, . \qquad (4a.21)$$

Thus, the cosmological red shift, RS equals

$$RS = [\gamma\underline{\lambda} - \underline{\lambda}]/\underline{\lambda} \qquad (4a.22)$$

Or by (1.7) and (4a.22)

$$RS = t/\mathbf{T} \tag{4a.23}$$

The distance d_e between O and any O' at time of emission, t = 0 at O equals ĉt. By (1.3) the constant recessional velocity, v* of O' relative to O equals

$$v^* = \hat{c}t/\mathbf{T}\gamma \tag{4a.24}$$

Or by (4a.23) and (4a.24) RS equals the ratio of v* and the light velocity, $\hat{c}/\gamma$ at any O' at time, t of reception, or

$$RS = v^*/[\hat{c}/\gamma] \tag{4a.25}$$

This is precisely the expression used presently in astronomy.

If the phase of light is emitted back in time at $t = -t_1$ at a distant origin O' then by (4a.1) the emitted wave length, $\lambda_e(-t_1)$ is given by

$$\lambda_e(-t_1) = \gamma(-t_1)\lambda(0) \tag{4a.26}$$

where $\lambda(0)$ is the wave length received at O. By (4a.21) the emitted wave length, $\lambda_e(-t_1)$ equals $\underline{\lambda}$, and

$$\lambda(0) = \underline{\lambda}/\gamma(-t_1) \tag{4a.27}$$

And the cosmological red shift, RS is given by

$$RS = [\underline{\lambda}/\gamma(-t_1) - \underline{\gamma}]/\underline{\lambda} \tag{4a.28}$$

or

$$RS = t_1/(\mathbf{T} - t_1) \tag{4a.29}$$

The distance between O' and O at time t = 0 is $\hat{c}t_1$. Hence by (1.3) the velocity of recession, v* of O' relative to O is

$$v^* = \hat{c}t_1/\mathbf{T}\gamma(-t_1) \tag{4a.30}$$

By (4a.29) and (4a.30)

$$RS = v^*/\hat{c} \tag{4a.31}$$

Eq. (4a.31) is precisely the expression used presently in astronomy. By (4a.29) one obtains the relation

$$1+RS = 1/\gamma(-t_1) \tag{4a.32}$$

The latter expression is used in Section 4f.

By (4a.21) the emitted wave length from any given kind of stable maser at any origin O' remote from large gravitating bodies equals a characteristic constant, $\underline{\lambda}$. The inertial light velocity, $\hat{c}/\gamma$ in the isotropic reference frame with origin, O' at time, t remains invariant with time for any given phase. Hence the frequency, $f_e(t)$ of the emitted phase is given by

$$f_e(t) = f_o/\gamma \tag{4a.33}$$

The constant, f_o is a characteristic constant for each kind of local stable masers. Atomic clocks advance at rates proportional to the emitted frequency, $f_e(t)$ of masers. Thus, local masers, or local atomic clocks, record local time, τ_L given by

$$d[\tau_L(t)]/dt = \gamma^{-1} \tag{4a.34}$$

or

$$\tau_L(t) = \mathbf{T} \ln \gamma \tag{4a.35}$$

In (4a.35) τ_L is synchronized to Newtonian time t, when t = 0.

Section 4b. One investigates next how Newton's orbital equations change due to the expansion effects. According to Newton's orbital equations the movements of n bodies under the action of their gravitational fields are determined by n equations, which for the body, j is

$$m'_j d^2[(H_i')_j]/dT^2 = -\sum_{q=1}^{n} k_o m'_q m'_j [(H_i')_j - (H_i')_q]/|(H_i')_j - (H_i')_q|^3 \tag{4b.1}$$

where m'_j and m'_q are respectively the constant masses of the bodies, j and q, $[H_i']_j$ and $[H_i']_q$ are respectively the mass center positions of the bodies, j and q relative to the common mass center of the n masses, and $q \neq j$ in the summation. The parenthesis (T) on H_i' and (t) on H_i are left out above and below for the sake of simplicity. By (3.8) and (3.16) the left side of (4b.1) for M is related as the corresponding quantity for N as

$$m'_j\, d^2(H_i')_j/dT^2 = m_j\, \gamma^{3} d^2(H_i)_j/dt \tag{4b.2}$$

By (3.8) and (4b.2) one finds that the expansion effects changes (4b.1) to

$$\gamma^{3} m_j d^2[(H_i)_j]/dt^2 = -\gamma^{2} \sum_{q=1}^{n} k_o m_q m_j [(H_i)_j - (H_i)_q]/|(H_i)_j - (H_i)_q|^3 \tag{4b.3}$$

Evidently, (4b.3) has the same form as (4b.1) when k is replaced by the gravitational function

$$k = \gamma^{-1} k_o \tag{4b.4}$$

A variation with time in the gravitational constant is precisely what is suggested in astronomy presently to eliminate the discrepancies between observed and calculated longitudes of planets of the solar system.

The ephemeredes are the calculated positions of the heavenly bodies as a function of mean solar time, t_s using Newton's orbital equations, (4b.1). Over centuries of observations discrepancies have been found to exist between the ephemeredes and the observed positions as a function of t_s. The cause of the discrepancies is in part the result of the frictions associated with lunar and solar tides, denoted tidal effects. These effects increase both the earth's sidereal rotational period and the moon's orbital period by different amounts. There is no measurable slow down in t_s due to tidal effects. However, by (4b.7) $t_s = t/\gamma$. The periods of observations are much less than $\mathbf{T}$, hence γ^{-1} is very nearly equal to $1-t/\mathbf{T}$, and by (4b.7)

$$t - t_s \approx t^2/\mathbf{T} \tag{4b.5}$$

Note that the slowdown in t_s relative to t has the parabolic form in t as observed, see for example [3, p. 190].

As mentioned above the ephemeredes do not agree with the observed positions of the planets and the moon. The general theory of relativity resolves some discrepancies, but not the ones that are studied below. The remaining discrepancies require empirical terms to be added to calculations to obtain the published Improved Ephemeredes. In [3] these empirical terms are eliminated to show the actual discrepancies between the ephemeredes and the observed positions of the planets and the moon. One of these is a discrepancy between the sun's calculated and observed mean longitude. This discrepancy is based on several centuries of observations and is known rather accurately. By [3, figs. 11.3 and 11.7] the discrepancy is about $1.23n^2$ seconds of arc over n centuries. The sun's orbital angular rate is one second of arc in about 24.4 seconds of time. Hence, the slow down in t_s relative to t over one century is given by

$$t - t_s = 30 \text{ sec} \tag{4b.6}$$

By (4b.5)

$$\mathbf{T} \approx t^2/(t - t_s) \tag{4b.7}$$

By (4b.6) and (4b.7) one obtains a value for $\mathbf{T}$ when for t^2 one insert the square of the number of seconds in one century, or about $9.96\ 10^{14}$ and when for $t - t_s$ one insert 30. This is a new means to calculate the value of $\mathbf{T}$, yielding the value $3.32 \cdot 10^{17}$ seconds or about 10.5 billion years. For this value $\mathbf{T}$ one finds by equation, (4a.2) that the light velocity diminishes with the progress of time, t by about 3 cm/sec per year. One suspects that the estimate of the discrepancy, $1.23n^2$ second of arc over n centuries may be improved by those involved in these observations and calculations. Eq. (4b.7) is then likely by far a more accurate means to determine $\mathbf{T}$ than those used presently.

Atomic time, τ determined by stable masers is by far the most accurate time scale. By (4a.35) when $|t| \ll \mathbf{T}$ the time difference, $t - \tau$ is very nearly given by

$$t - \tau = \tfrac{1}{2}\, t^2/\mathbf{T} \tag{4b.8}$$

By (4b.6) and (4b.8) one obtains

$$\tau - t_s = \tfrac{1}{2}\, t^2/\mathbf{T} \tag{4b.9}$$

By (4b.6) and (4b.9)

$$\tau - t_s = 15 \text{ sec} \quad (4b.10)$$

Section 4c. For M radioactive dating time, T_{rd} and Newtonian time, T are assumed to be identical. Therefore, for N motions radioactive dating time, Γ is related to Newtonian time, t as T is related to t in (1.4), or

$$\Gamma = t/\gamma \quad (4c.1)$$

The age of the universe is infinite on the time scales, Γ. Hence the ages of old stars and globular clusters in the Milky Way Galaxy determined by radioactive dating can exceed the Hubble age, **τ** by any amount. Thereby one resolves the "age paradox" that the age of old stars and the age of globular clusters in the Milky Way galaxy exceed credible estimates of the age of the universe. The difference between Γ and t is very small and the validity of (4c.1) likely cannot be established. There are no long time observations that establish this difference, like the long time observations and calculations of the difference between Newtonian time, t and solar ephemeris time, t_s that tend to establish the validity of (3.30) and (4b.7). In Section 4f one is able to show that (4c.1) is possible. However, one shows also there that radioactive dating time could just as well be on the atomic time scale, τ_L given by (4a.35).

Section 4d. By (4a.2) the differential local distance, dS moved relative to the expanding coordinates, x*'s by any phase of a light wave during the differential time, dt is

$$dS = \gamma^{-1}\, \hat{c}dt \quad (4d.1)$$

By (4a.35)

$$dS = \hat{c}\, d\tau_L \quad (4d.2)$$

By (4d.2) the local light velocity is constant and equal to $\hat{c}_i$, when it is reckoned with respect to local atomic time, τ_L. Hence local atomic time, τ_L play is equivalent to Einstein's proper time. However, τ_L is identical at all expanding coordinate points throughout the universe. It is no more a coordinate than t is in Newton's equation. To find an appropriate origin, O for the expanding coordinates is similar to finding an origin of inertial coordinates.

It is possible to determine how the expansion effects modify the equations of motion of the special of relativity. This is not done here because it is simpler first to use the present form of the equations, and then to correct the results by applying the appropriate multiplying factor containing the Hubble function, γ.

Section 4e. For N motions the gravitational energy, dE_g leaving via a weight less string used to lower a mass, Δm onto the surface of a spherical body of radius r_1 and mass $m(r_1)$ is very nearly

$$dE_g = k\, m(r_1)\, \Delta m \int_{\infty}^{r_1} (1/r^2)\, dr \quad (4e.1)$$

The mass, $m(r_1) = \rho_m\, (4/3)\pi(r_1)^3$. If $\Delta m = \rho_m 4\pi(r_1)^2 dr_1$ the radius r_1 is thereby increased by dr_1, and dE_g is given by

$$dE_g = k\,(16/3)\pi^2[\rho_m]^2(r_1)^4dr_1 \qquad (4e.2)$$

where k is the gravitational function (4b.4). The energy to build the central body from mass zero to mass, m of outer radius, r at any given time, t_1 is equal to the integral of (4e.2) from $r_1 = 0$ to $r_1 = r$, or

$$E_g(t_1) = 3k[m(r)]^2/5r \qquad (4e.3\ )$$

If instead the masses, Δm gradually fall onto $m(r_1)$ then the energy, $E_g(t_1)$ is released as thermal energy in m(r), called the energy of gravitational collapse. By (3.8), (3.32) and (4b.4)

$$E_g(t_1) = 3k_om^2/5r_o[\gamma(t_1)]^2 \qquad (4e.4)$$

where k_o and r_o are the values of k and r at time t = 0.

If the gravitational collapse takes place at a time, t_2, later than t_1, then by (4e.4) $E_g(t_2)$ is less, and less than $E_g(t_1)$. Therefore, the energy $E_g(t_1)$-$E_g(t_2)$ must be thermal energy generated in m between t_1 and t_2, or m generates thermal energy at a rate given by

$$dE_g/dt = (6k_om^2)/(5r_o\gamma^3\mathbf{T}) \qquad (4e.5)$$

Note that for M motions, i.e. $\gamma = 1$, the energy of gravitational collapse is independent of time, T of its occurrence. Thus equation (4e.5) is the rate of gravitational energy generated in all material bodies of constant density due to the pervasive Hubble expansion. Presently, at time t = 0, equation 4e.5) equals

$$dE_g/dt = (6k_om^2)/(5r_o\mathbf{T}) \qquad (4e.6)$$

By equation 4e.6) and with **T** equal to 10.5 billion years the amount of gravitational energy generated in one year equals about 5 billion times less than the energy of gravitational collapse, (4e.3). Moons, planets, stars, and galaxies have lifetimes of order five billion years. Thus the gravitational energy generated in such bodies over their life spans is about equal to the energy of gravitational collapse. Therefore, it is of importance to determine the temperature and thermal history of such bodies.

For the sun one must account for the variability in its density to determine its gravitational energy, $(E_g)_s$. Using the internal structure given in equation [9] one finds by numerical integration that $d(E_g)_s/dt$ is about

$$d(E_g)_s/dt = (9k_om_s^2)/(5r_s\mathbf{T}) \qquad (4e.7)$$

where m_s is the sun's mass and r_s its outer radius. If **T** equal to 10.5 billion years, then the value of (4e.6) is equal to $2.89{\cdot}10^{31}$ ergs/sec. The rate of total energy radiated by the sun is about $3.86{\cdot}10^{33}$ ergs/sec, or about 134 times larger than $d(E_g)_s/dt$, as it must be, since the sun's internal temperature is known to be due to nuclear reactions. However, if the sun runs out of nuclear fuel its temperature then may be determined by $d(E_g)_s/dt$. Thus, the magnitude of $d(E_g)_s/dt$ appears to be credible and not in conflict with present assumptions or observations. One notes also that if the sun runs out of nuclear fuel, $d(E_g)_s/dt$ is large enough to turn the sun into a red dwarf.

For the earth one must account for the variability in its density given in [10]. One finds by numerical integration that the rate of gravitational energy, $d(E_g)_e/dt$ released from the earth is about

$$d(E_g)_e/dt = [(7.1)k_o m_e^2]/(5r_e \mathbf{T}) \qquad (4e.8)$$

where m_e is the mass of the earth and r_e its outer radius. Using the value 10.5 billion years for $\mathbf{T}$, one finds that the value predicted by (4e.8) is $1.60\cdot10^{22}$ ergs/sec. By [11] the rate of heat conducted through the earth's crust is estimated to be $3.2\cdot10^{20}$ ergs/sec, or about 50 times less than (4e.8). Therefore, most of the heat loss from the earth's interior must occur as a result of eruptions of lava, hot water, and steam from below the continents and the ocean floors, which is then radiated to space. The heat intercepted by the earth from solar radiation is about $1.76\cdot10^{24}$ ergs/sec, which is about 110 times larger than (4e.8). If solar radiation were to cease the earth's average surface temperature would drop from about 273 ^oK to about 85^oK. Therefore, the earth's surface temperature is determined largely by the energy intercepted from solar radiation. Its interior temperature is largely determined by (4e.8) and possibly also by the energy released from atomic decay and other nuclear reactions. Thus, for the earth the magnitude of $d(E_g)_e/dt$ appears also to be credible. Because it is larger than the heat conducted through the earth's crust, as it must be since heat escapes also by the various kinds of eruptions, and it is considerably smaller than the energy intercepted by the earth from solar radiation, as it must be since this intercepted energy is known to determine the earth's surface temperature. The fact that both (4e.7) for the sun and (4e.8) for the earth fall within sensible limits is more remarkable since the ratio of the energies given by (4e.7) and (4e.8) is more than one billion. Moreover, the energy is determined by the gravitational constant, the Hubble age of the universe, and physical properties of the bodies.

The rates of gravitational energies generated in the outer planets are calculated below assuming that their densities are uniform. The rates of intercepted solar radiation are also calculated assuming that their absorptivities equal unity. All values are given in ergs/sec. The values given below are respectively $d[E_g]_p/dt$ for the planets and the rate of intercepted solar radiation.

Mars	$(2.85)10^{20}$	and	$(2.14)10^{23}$
Jupiter	$(1.21)10^{26}$	and	$(8.23)10^{24}$
Saturn	$(1.29)10^{25}$	and	$(1.74)10^{24}$
Uranus	$(7.02)10^{23}$	and	$(8.02)10^{22}$
Neptune	$(1.03)10^{24}$	and	$(2.93)10^{22}$

For Mars its surface temperature is determined by the rate of intercepted solar energy, and the rate of gravitational energy generated affects only its internal temperature. For the other planets the rates of gravitational energy generated affect both their surface and internal temperatures, and may also explain their low densities, atmospheric features, and also the “unseasonable” storms on Jupiter. One shows readily that for Pluto the gravitational energy is considerably less than the intercepted solar radiation, and the former energy affects only its internal temperature.

The mass of a gravitating body decreases because of the energy radiated. For the sun the rate of this mass loss, dm/dt presently equals the total energy radiated presently divided by c^2, or

$$dm/dt = (4.3)\cdot 10^{12} \text{ g/sec} \tag{4e.8}$$

Therefore, on the right side of (4e.4) one should include the term, $(6k_o m)(dm/dt)/5r_o$. The ratio of the latter term and the right side of (4e.4) is equal to $(\mathbf{T}/m)dm/dt$. For the sun this ratio equals about 1/1300. Thus, in (4e.6) the mass loss of the sun can be neglected. One shows readily that this is the case also for Newton's or relativistic orbital equations, i.e. the change in the orbital motion due to the expansion effects, which is equal to the discrepancies between the observed and calculated longitude of the sun, far exceeds those due to the mass loss of the sun.

If the sun's radius, r_s is replaced by the radius, r_1 to turn it into a Black Hole, then r_s/r_1 equals about $4.7\cdot 10^5$. In the absence of nuclear fuel the black hole would generate gravitational energy at a rate about 2300 times the sun's present rate due to nuclear reactions. The shrinking sun would generate gravitational energy at a rate proportional to $-dr_1/dt$ in addition to dE_g/dt, energies that likely would blow it apart long before it attains the radius, r_1. Therefore, the existence of Black Holes is rejected by the analysis of this paper.

Section 4f. By the above findings for the sun one expects that the sum of the energies generated internally by the stars of any galaxy is primarily the result of nuclear reactions, as long as nuclear fuel is available. By assumptions now in use in astronomy both the energy of atoms and their different energy levels are constants, and by the Planck constant. h_o the rate of energy, dE_g'/dT emitted by the thermally actuated atoms of standard galaxies is invariant with time of emission. The order of magnitudes of standard galaxies as functions of their red shifts, RS are then obtained as follows. The inertial coordinate to a galaxy emitting waves at time, $-t_1$ equals $-\hat{c}t_1$. Its constant velocity of recession, v^* is by (1.3) equal to $v^* = -\hat{c}t_1/(\mathbf{T}-t_1)$, and by (4a.31) $RS = v^*/\hat{c}$. Thus, by these two relations

$$RS = -t_1/[\mathbf{T}-t_1] \tag{4f.1}$$

The distance, d_R to a galaxy at time of reception, $t = 0$ of light emitted by the galaxy at time, $t = -t$ is $d_R = \hat{c}t_1 + v^*t_1$. Or by (4f.1)

$$d_R = \hat{c}\mathbf{T}t_1/(\mathbf{T}-t_1) \tag{4f.2a}$$

or by (4f.1) and (4f.2a)

$$d_R = \hat{c}\mathbf{T}\ RS \tag{4f.2b}$$

Therefore, the rate of energy per unit area, A received at time, $t = 0$ at the origin, O from any standard galaxy is

$$[dE_R'/dt]/A = \text{constant}\cdot(RS)^{-2} \tag{4f.3}$$

The order of magnitude, OM′ of the galaxy defined as

$$[dE_R'/dT]/A = 100^{-(1/5)OM'} \tag{4f.4}$$

Thus, OM′ becomes OM′ = constant + $5\cdot\log_{10}(RS)$. For standard galaxies the constant in OM′ has been determined to be about (20.5). Thus OM′ becomes

$$OM' = 20.5 + 5\cdot\log_{10}(RS) \tag{4f.5}$$

Eq. (4f.5) is the linear Hubble relation between $\log_{10}(RS)$ and OM′ for standard galaxies as shown in [5, fig. 5]. This equation is widely used in astronomy, and the above derivation is merely repeated here.

By (3.28) and (4a.18) the local energy, $[E(-t)]_L$ of any N motion at time $t = -t_1$ is given by

$$[E(-t)]_L = [E(0)]_L/[\gamma(-t)]^2 \tag{4f.6}$$

By (4a.32)

$$1+RS = 1/\gamma(-t_1) \tag{4f.7}$$

By (4f.6) and (4f.7)

$$[E(-t)]_L = [E(0)]_L/(1+RS)^2 \tag{4f.8}$$

Therefore, by (4f.1) the rates of energy emitted by distant standard galaxies with red shift, RS is larger by the factor, $(1+RS)^2$ than nearer standard galaxies at the present time. Thus, the expansion effects changes the ratio $[dE_R'/dT]/A$ to $[dE_R/dt]/A$ given by

$$[dE_R/dt]/A = \text{constant}\cdot(RS)^{-2}(1 + RS)^3 \tag{4f.6}$$

If OM is the order of magnitude as defined in (4f.4), one finds

$$OM = \text{constant} + 5\cdot\log_{10}[RS(1 + RS)^{-3/2}] \tag{4f.7}$$

The constant in (4f.7) is the same as the constant, 20.5 in (4f.5).

There is essentially no difference between OM′ and OM when RS is somewhat less than unity. As RS approaches and passes unity OM becomes completely different than OM′. Thus, OM reaches a maximum value of about 18.4 when RS equals 2, and above 2 the value of OM diminishes very slowly with increasing RS. When RS equals 4 then OM equals 18.3. The OM curve passes through the group of points for quasars shown in [5, fig. 5]. Thus, (4f.7) gives support to the widely held view that quasars are very distant galaxies and that their red shifts are cosmological or due to the Hubble expansion.

As reported in [4] and [5] quasars increase in number very rapidly for red shifts between ½ and 2, but none have been observed for red shifts beyond 4. By the discussion just prior to (4f.7) the rate of energy released by standard galaxies changes by the factor, $(1+RS)^3$, and for a red shift of 4 their emissions is larger by the factor, 125 than standard galaxies presently. This large energy release may preclude the existence of galaxies with red shifts lager than 4, as we now know them. The brightness and rather rapid variation in the emissions from quasars with red shifts above unity reported in [5] suggests that these quasars may be galaxies in their formative period. The spread in order of magnitudes of the quasars with red shifts between ¼ and 2 shown in [5, fig. 5] suggests that these quasars may be various kinds of galaxies at various stages of formation. During the formative period the rate of energy generated is likely greatly increased and variable with time as matter collapses to form stars and galaxies.

As mentioned above (4f.1), RS equals $t_1/(\mathbf{T}-t_1)$, and by (4c.1) one finds that

$$\Gamma(-t_1) = \mathbf{T}RS \tag{4f.8}$$

By (4f.1) and (4a.23) one obtains

$$\tau = - \mathbf{T} \ln (1 + RS) \tag{4f.9}$$

If the beginning of the formative period of the galaxies of the universe is between red shifts of 1 and 3, this corresponds by (4f.8) to a natural time, Γ between about minus 10.5 billion and minus 31.5 billion years, and by (4f.9) to an atomic time, τ between about 7.3 and 14.6 billion years. The ages of globular clusters and the oldest stars in the Milky Way galaxy, are estimated by radioactive dating to fall within either of these two ranges, see, for example [2] and [12]. Thus, radioactive dating time may possibly be on either of these two time scales, as was suggested in Section 4c. On the Newtonian time scale, t the corresponding time is by (4f.1) between ½ and ¾ of the Hubble age, $\mathbf{T}$ of the universe, or between 5.3 and 7.9 billion years.

In [7] and [8] one conclude from the high intensity received from Novae in very distant galactic neighborhoods that they are actually closer to us than indicated by the Hubble expansion of the universe for closer galaxies. Therefore, distances, d(0) in (I.2) is smaller and the Hubble constant H is for very distant galaxies larger than H for nearer ones. Therefore one concludes that the Hubble expansion is accelerating, and that the laws of physics may have to be amended. The observed great intensity of Novae in very distant galactic neighborhoods is precisely what one should expect from the difference between the modified Hubble relation, (4f.7) and the straight line Hubble relation, (4f.5). The observed great intensity of Novae in distant galaxies is due to the expansion effects (4a.18), which states that the energy of any N at time, $-t_1$ in the past has greater energy by the factor, $1/[\gamma(-t_1)]^2$ than the same N at time $t = 0$. Thus by the analysis in this paper the Hubble constant is the same for all galaxies and the Hubble expansion is not accelerating. In passing one notes that on the age of the universe is infinite on both the atomic and the planetary time scales. When the recessional velocities of the galaxies are reckoned with respect to either of these times the expansion of the universe is accelerating.

The derivation of OM in (5.9) is based on the ratio of rate of the photonic energy emitted at time, $t = -t_1$ and the photonic energy received at O at time, $t = 0$. Another way to derive OM for standard galaxies is to base it upon the ratio of the photonic energy emitted by distant emitters at time, $t = -t$ and the photonic energy emitted by identical emitters at the origin, O at time, $t = 0$. This ratio equals $[\gamma(-t_1)]^{-2}$. One shows readily then that that OM is replaced by OM* given by

$$OM^* = 20.5 + 5\cdot\log_{10}[RS/(1+RS)] \qquad (4f.10)$$

Both OM and OM* passes through the group of points observed for quasars. Thus, these observations cannot settle the question which of the latter two orders of magnitude is in accord with observations. The observed greater brightness of type 1a Novae in distant galaxies than expected from their distances away from us may be accurate enough to provide an answer to this question. The author feels that (4f.10) is the correct relation between the logarithm of RS and the order of magnitude for standard galaxies.

Brief Discussion. At the present time, $t = 0$ the Hubble factor, γ equals unity and there are then no differences between results obtained from any present theories of physics and those proposed in this paper. Also, the differences change with the progress of time, t by only about one part in 10.5 billion parts per year. Therefore, the differences are unobservable when calculations and observations extend over a decade or less. They begin to manifest themselves when calculations and observation extend over centuries. Yet these very minute changes with time resolve many outstanding questions and paradoxes that have puzzled and irritated scientists for many years. They increase without limits when calculations and observations extend back in time approaching the Big Bang, $t = -\mathbf{T}$.

The derivation of the cosmological red shift, RS used presently in astronomy is thought by many scientists to be a questionable application of Doppler's law. In Section 4a one is able to derive the identical expression for RS from fundamental principles.

The expansion effects change Newton's orbital equations simply by replacing the gravitational constant, k_o by the gravitational function, $k(t) = k_o/\gamma$, which diminishes with the progress of time, t. This is of interest since a decrease with time in the gravitational constant is of magnitude equal to what is proposed presently in astronomy to eliminate the well known discrepancies between the observed and calculated longitudes of the sun. Moreover, these discrepancies provides a new independent means, (4b.7) to determine the Hubble age $\mathbf{T}$ of the universe, yielding the value 10.5 billion years. Thereby, all other expansion effects are determined.

If the half life of radioactive materials is invariant with time, as assumed presently in physics, then by the expansion effects radioactive dating time and planetary ephemeris time falls on the same time scale, Γ equal to t/γ. Since the age of the universe is infinite on the time scale, Γ the ages of globular clusters and old stars in the Milky Way galaxies determined by radioactive dating can exceed the Hubble age, $\mathbf{T}$ on the time scale, t by any amount. Thus, one resolves the "age paradox" that by some estimates the ages of globular clusters and old stars exceed the age of the universe.

Equation (4e.2) gives the energy of gravitational collapse released when a gravitating body is formed by matter gradually falling onto it, and thus gradually increasing its radius from zero to its final value. Eq. (4e.5) gives the rate of energy generated in all material bodies due to the expansion effects. The energy in (4e.5) generated in one year is 5 billon times smaller than the energy in (4e.2). Like all expansion effects they occur at exceedingly small rates. If the age of the body equals the age of the earth, estimated to be of order 5 billion years, then the energy in (4e.5) generated over the life of the body equals the energy in (4e.2). The latter energy is important primarily during the formation of the earth, the former is apt to be important at the present time. However, both energies must be taken into account to estimate the temperature and thermal history of the earth and all other heavenly bodies of the universe.

In the paper one gives two possible relations between the logarithm of red shifts and orders of magnitude for standard galaxies. They are modifications of the straight line Hubble relation between the logarithm of red shifts and orders of magnitude of standard galaxies. The modifications do not manifest themselves until the red shift nears and passes unity. Like all expansion effects they are zero presently, $t = 0$ and changes very slowly with the distance back in time. When the red shift approaches and passes unity the both modified Hubble relations pass through a group of points observed for quasars. Thus either of these two expansion effect supports the widely held view that quasars are distant galaxies. The greater brightness of type 1a Novae expected from their distances away from us, as determined by Hubble's law, may be sufficiently accurate to determine which of the two possible modifications of the straight line Hubble relation are apt to be correct.

In Appendix A one discusses the possibility that vortex tubes or filaments may be part of the physical makeup of photons, electrons, positrons, strings, and nucleons. Moreover, one gives reasonable argument that the density of the aether is of order equal to the nuclear density. The very dense aether serves as an omnibus in which ride all forms of matter, fields, and propagations in the universe.

In Appendix B the forward motion of perihelia and gravitational deflections and red shifts of light are obtained by a reexamination of an extension [15] of the equations of motion of the special theory of relativity describing test particles moving in Schwarzschild fields. One finds that the former of the two effects predicted by the Schwarzschild solutions are too large by 20 percent. The gravitational red shift is correct. One shows that the pervasive Hubble law does not alter forward motions of perihelia per orbital revolutions and gravitational deflections and red shifts of light, but it changes the orbital radii and orbital periods by the factor, γ.

In Appendix C one discusses major mechanisms causing tectonic plate movements and seafloor spreading.

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Appendix A. The propagating circulation singularities described in [15] satisfy the nonlinear Euler's equations for a specific ideal fluid and also the linearized Euler's equations. Below one describes a combination of two such propagating circulation singularities that satisfy the linearized Euler's equations. They may be fluid motions, M which describe the nature of a photon or a single light wave propagating in the stationary aether along the X_3 axis. The wave has the velocity potential, φ' that is a function of X_1, X_2, and X_3-$\hat{c}T$, namely,

$$\varphi'=\hat{c}A\{\tan^{-1}[X_1-A\alpha(\theta)]/[X_2-A\beta(\theta)]-\tan^{-1}[X_1+A\alpha(\theta)]/[X_2-A\beta(\theta)]\} \qquad (A.1)$$

where $\hat{c}$ is its constant velocity of propagation, A is a constant of dimension length, and α and β are non dimensional functions of θ given by

$$\theta = (X_3 - \hat{c}T)/\lambda_o \qquad (A.2)$$

where λ_o is a constant equal to its wave length, X_3-$\hat{c}T$ varies from zero to λ_o, and θ varies from zero to unity. The components of the fluid velocity, v_i' equal to $-\partial\varphi'/\partial X_i$ are

$$v_1' = \hat{c}\{-A[X_2-A\beta(\theta)]/(h_1)^2 + A[X_2-A\beta(\theta)]/(h_2)^2\} \qquad (A.3)$$

$$v_2' = \hat{c}\{A[X_1-A\alpha(\theta)]/(h_1)^2 - A[X_1+A\alpha(\theta)]/(h_2)^2\} \qquad (A.4)$$

$$v_3' = \hat{c}A/\lambda_o\{\{[-(X_1-A\alpha(\theta)][Ad\beta(\theta)/d\theta)]+[X_2-A\beta(\theta)][Ad\alpha(\theta)/d\theta)]\}/(h_1)^2 +$$

$$+ \{[X_1+A\alpha(\theta)][Ad\beta(\theta)/d\theta] +[X_2-A\beta(\theta)][Ad\alpha(\theta)/d\theta]\}/(h_2)^2\} \qquad (A.5)$$

where

$$(h_1)^2 = [X_1-A\alpha(\theta)]^2 + [X_2-A\beta(\theta)]^2 \qquad (A.6)$$

$$(h_2)^2 = [X_1+A\alpha(\theta)]^2 + [X_2-A\beta(\theta)]^2 \qquad (A.7)$$

The wave has two lines of propagating circulation singularities, namely, the line

$$X_1 = A\alpha(\theta)\ ,\ X_2 = A\beta(\theta) \qquad (A.8)$$

with left handed circulation about the axis of propagation, and the line

$$X_1 = -A\alpha(\theta)\ ,\ X_2 = A\beta(\theta) \qquad (A.9)$$

with right handed circulation about the axis of propagation of equal strength. Both lines start on the X_3 axis at the point, X_3 equal to $\hat{c}T$ and end on the X_3 axis at the point, X_3 equal to $\hat{c}T + \lambda_o$. Each line has tangents parallel to the X_3 axis at both end points. Thereby, $\alpha(\theta)$, $\beta(\theta)$, $d\alpha(\theta)/d\theta$, and $d\beta(\theta)/d\theta$ all vanish at the end points, or at θ equal to zero and θ equal to unity. By (A.3) to (A.5) the fluid velocity v_i vanish on the planes, X_3 equal to $\hat{c}T$ and X_3 equal to $\hat{c}T + \lambda_o$. The wave's activity region is confined between these two planes. For any given functions $\alpha(\theta)$ and $\beta(\theta)$ the constant, A defines the distance of the two lines of circulation singularity from the X_3 axis. One shows readily that

$$\partial^2\varphi'/\partial X_1^2 + \partial^2\varphi'/\partial X_2^2 \equiv 0 \qquad (A.10)$$

and

$$\partial^2\varphi'/X_3^2 = (1/\hat{c}^2)\partial^2\varphi'/\partial T^2 \qquad (A.11)$$

or that

$$\partial^2\varphi'/\partial X_i^2 = (1/\hat{c}^2)\partial^2\varphi'/\partial T^2 \qquad (A.12)$$

Thus, φ' satisfy Euler's linearized equations, (A.12) for ideal fluids. This is true for any choice of A and functions, $\alpha(\theta)$ and $\beta(\theta)$. Therefore, the linear fluid dynamic equations are satisfied for any choice of lines of circulation singularities that satisfies the above mentioned end conditions. If such lines can be found so that φ' also satisfies Euler's nonlinear equations for the adiabatic monatomic ideal fluid remains to be seen. One may require additional functions with activity region confined between the two planes, which possibly can be obtained by numerical means. To change the stationary aether to the expanding aether, R one simply superimpose the expanding effects.

The most interesting aspect of such waves is that the "spikes" at each end of the two lines of circulation, (A.8) and (A.9) allow the circulation singularities to slip through the stationary aether without pulling any part of it along. At the front end of the wave the aether on the side of the X_2 X_3 coordinate plane where X_1 is positive is turned into the left handed circulation path, and on the side where X_1 is negative it is turned into the right handed circulation path of equal strength. At the rear end of the wave both circulations cancel each other as they slide off their paths leaving no wake behind the wave. If such waves can be found that satisfies Euler's nonlinear equations, they would have zero rest mass. Like other wave forms they transmit energy through the fluid without pulling the fluid along.

Pair productions, where very energetic photons near charged matter are converted to electron-positron pairs, suggest the possibility that electrons and positrons may be made up of the two closed line propagating circulation singularities for a photon described in (A.1) to (A.9). Such closed line propagating circulation singularities would have spins. According to Kelvin's circulation theorem and corollaries, see for example [17], the strengths of circulation singularities must remain invariant with time, and these fluid motions would maintain their characteristics over eons of time.

The fluid making up the closed line of circulation singularities for electrons and positrons must remain invariant, and their rest masses, m_r' and m_r likely reside nearly entirely within the volumes, $V_c'(T)$ and $V(t)$ enclosing the singularities, or

$$m'(T) = \iiint_{V'(T)} \rho' dV' \qquad (A.13)$$

$$m(t) = \iiint_{V(t)} \rho dV \qquad (A.14)$$

Hence, the densities, ρ_o and $\rho^*(0)$ of the stationary and expanding aether must be near the nuclear density of order 10^{12} g/cm^3. Neither ρ nor $V(t)$ change in passing from an inertial coordinate point, x_i to a coinciding local coordinate point x_i^*. Hence, the inertial and local rest masses of material bodies. This is unlike the inertial and local photonic masses of a light wave which are different, as mentioned in Section 4a. Relativistic effects come about when these rest masses move through the aether. As their velocities increase more fluid motions outside their rest masses become associated with them. As a result their masses increase, and the increases in their masses come from the density of the aether.

The purpose of this Appendix is to show that there are possible fluid motions that may make up light and matter. If such motions are found, satisfying (1.8) to (1.10), they are likely entirely different than those described here.

Appendix B. In [13] the three Einstein effects are obtained by an extension of the equations of motion of the special theory of relativity to test particles moving in a Schwarzschild gravitational field. For the general theory of relativity see, for example, [14]. In this Appendix [13] is reexamined with the assumption that test particles, the Schwarzschild field, and the central mass are interacting M motions superimposed on the stationary aether.

The governing equations, [13, Eq's (1.1) to (1.4)] are restated below as they appear in [13] with one exception. Namely, equation [13, Eq.(1.4)] is replaced by its time derivative, [13, Eq.(1.5)]. The reason for this is simply that in [13] and in the present analysis one uses only the time derivative of [13, Eq.(1.5)]. Equation [13, Eq.(1.5)] is identical to (B.4) below.

$$\mathbf{F} - (\mu m/r^3)\mathbf{r} = d(m\mathbf{v})/dt \quad \text{(B.1)}$$

$$m = m_o'[1 - (v/c')^2]^{-1/2} \quad \text{(B.2)}$$

$$E = m(c')^2 \quad \text{(B.3)}$$

$$dE/dt = \mathbf{v}\cdot\mathbf{F} + (\mu m/r^2)dr/dt \quad \text{(B.4)}$$

One uses the notation of [13], where the primed quantities, c' and m_o' are in turn the light velocity and the test particle's rest mass, which are functions of the orbital radius, r. Their constant values remote from the central mass are respectively $\hat{c}$ and m_o. The constant, μ is the product of the gravitational constant, k_o and the mass of the massive central body. Bold face letters are vectors and the same ordinary face letters are their magnitudes. Any non gravitational force acting on the test particle is denoted **F**. The vectors, **r** and **v** are in turn the orbital radius vector and orbital velocity of the test particle, and m is its mass and E is its energy. In passing one notes that in [13, Eq.(1.4)] the constant rest energy, $m_o\hat{c}^2$ of the test particle remote from the central mass is omitted. However, as mentioned above, only its time derivative, [13, Eq.(1.5)], or equivalently (B.4), is employed. Therefore the omission is of no consequence. The first three equations require no further explanations. In [13] the fourth equation and its last term, $(\mu m/r^2)dr/dt$ are essentially "pulled out of a hat" to obtain the three Einstein effects. The major subject matter of this paper is to explain the meaning of (B.4).

Equations (B.1) to (B.4) are six scalar equations for the determination of seven unknowns, m, E, c', m_o', and the three scalar components of **v**. Without any assumptions in addition to (B.1) to (B.4), an additional equation is obtained in [13] by a procedure repeated below in a slightly different manner. Namely, a multiplying constant, q is inserted in the last term of (B.4), or equation B.4) is replaced by

$$dE/dt = \mathbf{v}\cdot\mathbf{F} + q(\mu m/r^2)dr/dt \qquad (B.5)$$

By simple manipulations one obtains from (B.3)

$$2m\, dE/dt = m^2 d(c')^2/dt + d(mc')^2/dt \qquad (B.6)$$

By (B.2), (B.5), and (B.6) one obtains

$$2m\mathbf{v}\cdot\mathbf{F}+(2q\mu m^2/r^2)dr/dt = m^2 d(c')^2/dt + d(m_o'c')^2/dt + d(mv)^2/dt \qquad (B.7)$$

Scalar multiplication of (B.1) by 2m**v** yields

$$2m\mathbf{v}\cdot\mathbf{F} = (2\mu m^2/r^2)dr/dt + d(mv)^2/dt \qquad (B.8)$$

When (B.7) is inserted in (B.8) one obtains

$$m^2[d(c')^2/dt + 2(q+1)(\mu/r^2)dr/dt] = - d(m_o'c')^2/dt \qquad (B.9)$$

Since c' and m_o' are by assumptions functions of r, the two time derivatives, $d(c')^2/dt$ and $d(m_o'c')^2/dt$ can be replaced by $[d(c')^2/dr]dr/dt$, and $d[(m_o'c')^2/dr]dr/dt$, and dr/dt cancels out. Thus, (B.9) becomes

$$m^2 d[(c'^2) - 2(q+1)\mu/r]/dr = - d(m_o'c')^2/dr \qquad (B.10)$$

The right side of (B.9) is a function of r, but its left side is not. At any given position, **r** the mass, m may have any value depending on the history of the force, **F** acting on the test particle. Hence, both sides of (B.10) must be zero. This circumstance provides the additional scalar equation necessary to determine the seven unknowns mentioned above. In passing one note that there is a misprint in sign in [13, Eq.(1.8)]. It does not carry forward, since both sides of the equation are zero.

By (B.8) the light velocity, c' and rest mass, m_o' of the test particle are

$$c' = \hat{c}\Phi^{1/2} \qquad (B.11)$$

$$m_o' = m_o\Phi^{-1/2} \qquad (B.12)$$

where

$$\Phi = 1 - 2(q+1)\mu/r\hat{c}^2 \qquad (B.13)$$

By (B.11) and (B.12) the rest energy, $E_o' = m_o'(c')^2$ is given by

$$E_o' = m_o\hat{c}^2\,\Phi^{1/2} \qquad (B.14)$$

In the solar system $\mu/r\hat{c}^2$ is less than or equal to about $2\cdot10^{-7}$ and Φ^n is sufficiently accurately given by

$$\Phi^n \approx 1 - 2n(q+1)\mu/r\hat{c}^2 \qquad (B.15)$$

Terms of higher order in $\mu/r\hat{c}^2$ produce no measurable effects on the test particle's motion, and (B.15) with q equal to unity yields the three Einstein effects, as shown in [13]. When higher order terms in μ/rc^2 are neglected in the analysis below, the equal sign is replaced by the "nearly equal sign", ≈. Thus,

$$E_o' \approx m_o\hat{c}^2 - (q+1)\mu m_o/r \qquad \text{(B.16)}$$

If the test particle is slowly lowered from infinity to the radius, r by a weightless string the radial force, F in the string is by (B.12) equal to $(\mu m_o\Phi^{-1/2})/r^2$, which also can be written as

$$F = [m_o\hat{c}^2/(q+1)]\ d(\Phi^{1/2})/dr \qquad \text{(B.17)}$$

The work, W done by F as the test particle is slowly lowered from infinity to the radius, r is

$$W = -[m_o\hat{c}^2/(q+1)][\Phi^{1/2}(\infty) - \Phi^{1/2}(r)] \qquad \text{(B.18)}$$

or

$$W = -\ m_o\hat{c}^2(1-\Phi^{1/2})/(q+1) \qquad \text{(B.19)}$$

or

$$W \approx -\ m_o\mu/r \qquad \text{(B.20)}$$

The work, W is energy removed from the rest energy of test particle via the weightless string. One notes also that W is equal to the potential of the gravitational force. By (B.16) the change in the energy of the rest mass between infinity and r is equal to $-(q+1)\mu m_o/r$. Thus, only the part, $-\ m_o\mu/r$ leaves the rest energy via the string. The other part, $-q\mu m_o/r$ must leave the rest energy by a different process. There are several fundamental reasons for the view that the latter loss is due to the change in the fluid dynamic potential energy, P_f of the test particle. First, by the fundamental thesis of this paper the test particle must have fluid dynamic potential energy. Second, the additional energy change is not caused by external forces on the test particle, because the force, F in the string cancels the gravitational force as indicated above. Thus, P_f is like the fluid dynamic potential energy, $Pd\upsilon$ of a fluid element, an energy that also can change without external forces acting on the mass element, $\rho d\upsilon$. Second, the fluid pressure must diminish as r diminishes, since the gravitational force is attractive. Hence, P_f must diminish as r diminishes, as indicated above. Thus, the last term in (B.4) or (B.5), which in [13] was essentially "pulled out of a hat", turns out to be the test particles fluid dynamic energy, P_f, given by

$$P_f = -\ m_o\hat{c}^2(1 - \Phi^{1/2})/(q+1) \qquad \text{(B.21)}$$

or

$$P_f \approx -q\mu m_o/r \qquad \text{(B.22)}$$

Moreover, P_f is not caused by work of the gravitational force, but by the drop in the fluid pressure as r diminishes. In accord with Einstein all energy have mass, and P_f adds mass to the test particle in the amount, $qm_o\mu/r\hat{c}^2$. Thereby it increases the gravitational force by a very small amount compared to $\mu m_o/r$.

When **F** ≡ 0, by (B.3), (B.5), and (B.11), (B.6) can be written as

$$d(\ln E)/dt = [q/2(q+1)]d(\ln\Phi)/dt \tag{B.23}$$

or

$$d(\ln E)/dt = d(\ln\Phi^{q/2(q+1)})/dt \tag{B.24}$$

Thus, E can be written as

$$E = \bar{A} m_o \hat{c}^2 \Phi^{q/2(q+1)} \tag{B.25}$$

The constant energy $m_o\hat{c}^2$ is included in (B.25) to make Ā non-dimensional constant. From (B.2), (B.3), (B.11), (B.12), and (B.25)

$$\bar{A} = \Phi^{1/2(q+1)}[1 - (v/c')^2]^{-1/2} \tag{B.26}$$

By (B.26), the value of Ā is approximately given by

$$\bar{A} \approx 1 + (½)(v/\hat{c})^2 - \mu/r\hat{c}^2 \tag{B.27}$$

or

$$(\bar{A} - 1)\hat{c}^2 \approx (½)v^2 - \mu/r \tag{B.28}$$

Equations (B.28) is the vis viva integral. One shows readily that for hyperbolic and parabolic orbits the constant, Ā equals respectively, $1+(½)(v_\infty/\hat{c})^2$ and 1, where v_∞ is the velocity of the test particle at infinity. For elliptic orbits

$$\bar{A} = 1 - \mu/2\bar{a}\hat{c}^2 \tag{B.29}$$

where the constant, ā is its semi- major axis. By (B.28) and (B.29)

$$(½)v^2 - \mu/r = - \mu/2\bar{a} \tag{B.30}$$

This is the vis viva integral for elliptic orbits. It makes the radii of perigee and apogee of the test particle invariant with time, which is in accord with the Schwarzschild solution of the general theory of relativity.

Solutions of (B.1), (B.2), and (B.3), i.e. q equal to zero, predicts a forward motion of perigee and a gravitational deflection of light that is only half the values observed. Einstein was familiar with this solution. To correct its deficiencies, Einstein introduced the notion of curved space, a notion not found in any other laws of physics. In this paper one introduce the notion that the test particle must have fluid dynamic energy given by (B.21) or approximately by (B.22).

By (B.25) and (B.26)

$$E = m_o\hat{c}^2\Phi^{1/2}[1 - (v/c')^2]^{1/2} \tag{B.31}$$

or

$$E \approx m_o\hat{c}^2 [1 - (q+1)\mu/r\hat{c}^2 + (½)(v/\hat{c})^2] \tag{B.32}$$

or by (B.30)

$$E \approx m_o\hat{c}^2\ [1 - \mu/2\bar{a}\hat{c}^2 - q\mu/r\hat{c}^2] \qquad (B.33)$$

The energy, E is the sum of the test particles kinetic energy, the potential of the gravitational force, and its fluid dynamic potential energy. Thus, (B.33) shows that the sum of the former two energies remains nearly invariant with time. The only variable term is P_f equal to $-qm_o\mu/r$, and, with **F** equal to zero, one note that (B.5) is satisfied.

The value of q can be deduced as follows. For any M motion the force, df'_g acting on a fluid mass element, $\rho'dv'$ at any point X_i at time, T is by (1.11) given by

$$df'_g(X's,T) = dv'(X's,T)\ d[p'(X's,T)]/dX_i \qquad (B.34)$$

By (1.13) it can also be written as

$$df'_g(X's,T) = dv'(X's,T)\hat{c}^2[\rho'(X's,T)/\rho_o]^{2/3} \qquad (B.35)$$

The change in the fluid dynamic potential energy, dP'_f of the same fluid mass element is given by

$$dP'_f(X's,T) = p'[(X's,T)]d[dv'(X's,T)]/dX_i \qquad (B.36)$$

By (1.13) it can be written as

$$dP'_f(X's,T) = -\ (3/5)\ dv'(X's,T)\hat{c}^2[\rho'(X's,T)/\rho_o]^{2/3} \qquad (B.37)$$

By (B.35) and (B.37) the change in P'_f should equal –(3/5) times the gravitational force, or q should equal (3/5).

The motion of the perihelion of Mercury is cumulative and can be determined rather accurately to be 43 seconds of arc per century. R. Dicke has proposed that about 10 percent of the observed seconds of arc, or 4.3 arc seconds, may be due to the sun's oblateness; see for example [16]. By the findings above about 8.6 seconds of arc per century must be due to the sun’s oblateness, and the amount predicted by Schwarzschild solution may be too large.

The gravitational deflection of light at grazing incidence of the sun is difficult to measure. By the above analysis it should be 20 percent less than the value predicted by Einstein, or about 1.4 seconds.

The value of the factor, q has no effect on the gravitational red shift, which is shown as follows. Eq. (B.24) applies for light waves and yields for the photonic energy, E_p of light

$$E_p \approx (E_p)_\infty \Phi^{q/2(q+1)} \qquad (B.38)$$

The subscript, ∞ indicates the value of E_p remote from the central mass. Because of the static character of the Schwarzschild field, the frequency, f of any given light ray has the same value when observed at any fixed coordinate position, **r**, see for example the paragraph below [14, Eq.(14.48)]. Hence, by the Planck's relation, $E_p = h_o f$ and (B.38) the Planck constant, must be replaced by the function, $\hat{h}$ given by

$$\hat{h} \approx h_o\, \Phi^{q/2(q+1)} \tag{B.39}$$

where h_o is the Planck constant at infinity for M motions. The energy, E_a of atoms at any fixed coordinate position, **r** is by (B.13) given by

$$E_a = (E_a)_\infty\, \Phi^{1/2} \tag{B.40}$$

and the difference, ΔE_a between their various energy levels is given by

$$\Delta E_a = (\Delta E_a)_\infty \Phi^{1/2} \tag{B.41}$$

The frequency, f emitted by a maser at any fixed position, **r** is given by

$$\Delta E_a = \hat{h} f \tag{B.42}$$

or

$$f \approx [(\Delta E_a)_\infty \hat{h}_\infty^{-1}]\, \Phi^{1/2(q+1)} \tag{B.43}$$

or

$$f \approx f_\infty (1 - \mu/r\hat{c}^2) \tag{B.44}$$

Thus, the red shift, $1 - f/f_\infty = \mu/r\hat{c}^2$ is the same for all values of q.

The above analysis to explain the three Einstein effects is important since it is tied to a unified theory, namely, Euler's equations for the monatomic adiabatic ideal fluid. Moreover, one shows that Newton's three simple laws, governing the movements of the fluid elements making up matter, fields, and light, reveal themselves on the larger level of the movements of material bodies, like the heavenly bodies of the solar system, in similar but more complex and approximate laws, namely, the extended equations of motion of the special theory of relativity, (B.1) to (B.4).

The above analysis describes interacting M motions, which are deficient, like the Schwarzschild solution, since they do not account for the expansion effects. To remove this deficiency the expansion effects must be superimposed. By (3.8), (3.32), and (4b.4) the expansion effects do not alter the value of term $\mu/r\hat{c}^2$. Thus, the forward motion of the perihelion per orbital period, the gravitational deflection of light, and the gravitational red shift are not altered by the expansion effects. However, the expansion effects cause orbital radii and orbital periods to change with the progress of time, t by the factor, γ.

Appendix C. The fluid dynamic potential energy, P_f, described in Appendix B, is very likely a main ingredient in the mechanics of plate tectonics. By (B.22), (B.35) and (B.37)

$$P_f \approx -3\mu m_o/5r \tag{C.1}$$

where μ equals the product of the gravitational constant and the sun's mass, m_o is the rest mass of the earth and r its orbital radius. As discussed below (B.33) the sum of the earths kinetic energy and the potential energy of the gravitational force acting on the earth are equal and opposite. Therefore, P_f is the only variable energy of the earth as it orbits the sun. By Einstein's relation, $E = mc^2$, it causes the mass of the earth to vary an amount, Δm given by

$$\Delta m \approx -(3\mu m_o)/(5c^2 r) \qquad \text{(C.2)}$$

At perigee and apogee $r = r_o(1\text{-}e)$ and $r_o(1+e)$ respectively, where r_o is the semi major axis and e is the eccentricity of the earth's orbit. If m_p and m_a are the earth's masses at perigee and apogee respectively then

$$m_a - m_p = (6\mu m_o e)/[5c^2 r_o(1\text{-}e^2)] \qquad \text{(C.3)}$$

By (C.3) $m_a - m_p$ equals about (1.118) 10^{18} g. It is equivalent to the mass of a layer of crust over the earth's surface of thickness, $t_c \approx (0.081)$ cm.

Since the earth is compressible its radius will be smallest at apogee and largest at perigee. The difference between the two radii is likely considerably less than the thickness, t_c. Therefore, one can readily show that the mass and radius changes will have only a negligible effect on the known seasonal change in the length of day. However, the radius change of the earth, which remains to be estimated, is likely a major contributor to tectonic plate movements and seafloor spreading. One notes also that expansion induced thermal energy (4e.5) is another important contributor to tectonic plate movements. It causes a very slow continuous increase in the size of the earth until possibly some state of equilibrium is reached. The heat generated below the surface of the earth is independent of geographical positions. The manner in which it gets to the surface of the earth is likely to be different under continents than under oceans.